UNIT IV

FAULT ANALYSIS - UNBALANCED FAULT

(1)

Introduction :-

The unsymmetrical faults are the faults in which the fault currents in the 3 phases are unequal. The different types of unsymmetrical faults are

1) Single line to grnd fault

2). Line to line fault

3) Double line to gnd fault

4) One or two open conductor faults.

Since any unsymmetrical fault causes unbalanced currents to flow in the s/m, the unsymmetrical faults are analyzed using symmetrical components.

Symmetrical Components :

The analysis of unsymmetrical polyphase n/w by the method of symmetrical components was introduced by Dr. C. Fortescue.

An unbalanced s/m of n related vectors can be resolved into n s/m of balanced vectors called symmetrical components of original vectors.

The n vectors of each set of components are equal in length and the phase angles b/w adjacent vectors of the set are equal.

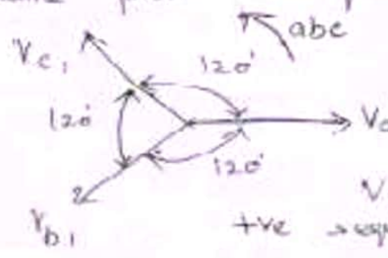
In a 3 ϕ s/m, the three unbalanced vectors (either V_a, V_b, V_c or I_a, I_b & I_c) can be resolved into 3 balanced s/m of vectors. The vectors of the balanced s/m are called symmetrical components of the original s/m. The sym. components of 3 ϕ s/m are

(i) Positive Sequence Components

(ii) Negative Sequence Components

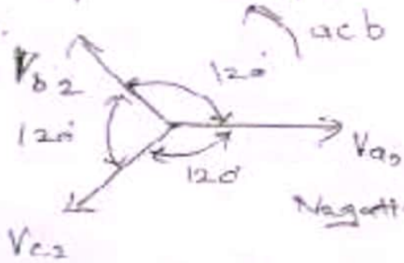
(iii) Zero Sequence Components

* The Positive sequence components consists of 3 vectors equal in magnitude, displaced from each other by 120° in phase, & having the same phase sequence as the original vectors.



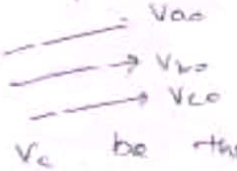
Vector diagram
+ve sequence Components

* The negative sequence components consists of three vectors equal in magnitude, displaced from each other by 120° in phase, and having the phase sequence opposite to that of the original vectors.



Vector diagram
Negative seq. components

* The zero sequence components consists of three vectors equal in magnitude and with zero phase displacement from each other.



Vector diagram
Zero seq. components

Let V_a, V_b, V_c be the set of unbalanced voltage vectors with phase sequence abc. Each voltage vector can be resolved into positive, negative and zero sequence components.

Let V_{a1}, V_{b1}, V_{c1} → +ve seq. components of V_a, V_b, V_c resp. with phase sequence abc.

V_{a2}, V_{b2}, V_{c2} → -ve seq comp of V_a, V_b, V_c resp. with phase sequence acb

V_{a0}, V_{b0}, V_{c0} → zero seq. components of V_a, V_b, V_c res.

From the vector diagrams of sym. components the following conclusions can be made.

1). On rotating the vector V_{a1} by 120° in anticlockwise direction we get V_{c1}.

2). On rotating the vector V_{a1} by 240° in anticlockwise direction we get V_{b1}.

3). On rotating V_{a2} by 120° → anti → V_{a2}.

4) " V_{a2} " 240 " → V_{c2}.

We can define an operator which causes a rotation of 120° in the anticlockwise direction, such an operator is denoted by the letter 'a'.

$$a = 1\angle 120^\circ = 1\, e^{+j2\pi/3} = \cos\frac{2\pi}{3} + j\sin\frac{2\pi}{3}$$

$$= -0.5 + j0.866$$

$$a^2 = 1\angle 240^\circ = -0.5 - j0.866$$

$$a^3 = 1\angle 360^\circ = 1$$

$$1 + a + a^2 = 1 + (-0.5 + j0.866) + (-0.5 - 0.866j)$$

$$= 0$$

Computation of Unbalanced vectors from their symmetrical components.

Each of the original vector is the sum of its positive, negative and zero sequence components. Therefore the original unbalanced three phase voltage vectors can be expressed in terms of their components as shown below.

$$V_a = V_{a0} + V_{a1} + V_{a2} \quad \ldots \quad (1)$$

$$V_b = V_{b0} + V_{b1} + V_{b2} \quad \ldots \quad (2)$$

$$V_c = V_{c0} + V_{c1} + V_{c2} \quad \ldots \quad (3)$$

From the vector diagrams, we get the following relations b/w various symmetrical components.

$$V_{b0} = V_{a0}\ ; \quad V_{b1} = a^2 V_{a1}\ ; \quad V_{b2} = aV_{a2} \ldots (4)$$

$$V_{c0} = V_{a0}\ ; \quad V_{c1} = aV_{a1}\ ; \quad V_{c2} = a^2 V_{a2} \ldots (5)$$

Using eqns 4, 5, 5, the eqns 1, 2, 3 can be written as

$$V_a = V_{a0} + V_{a1} + V_{a2} \quad \text{---} \quad (6)$$

$$V_b = V_{a0} + a^2 V_{a1} + a V_{a2} \quad \text{---} \quad (7)$$

$$V_c = V_{a0} + a V_{a1} + a^2 V_{a2} \quad \text{---} \quad (8)$$

In matrix form,

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} \quad \text{---} \quad (9)$$

The eqn 9 can be used to compute the unbalanced voltage vectors from the knowledge of sym. components.

Computation of sym. components of unbalanced vectors:

The matrix eqn 9 can be written in the vector notation as

$$V = A\, V_{sy} \longrightarrow (10)$$

where $V = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$; $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$; $V_{sy} = \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$

On premultiplying the eqn 10 by A^{-1}

$$A^{-1} V = A^{-1} A\, V_{sy}$$

$$V_{sy} = A^{-1} V \longrightarrow (11)$$

$$A^{-1} = \frac{\text{Adjoint of } A}{\text{Determinant of } A}$$

Let Δ = Determinant of A

$$\therefore \Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix} = 1\,(a^4 - a^2) - 1\,(a^2 - a) + 1\,(a - a^2)$$

$$= (a - a^2) + (a - a^2) + (a - a^2)$$

$$= 3\,(a - a^2)$$

$(\because a^4 = a^3 \cdot a = a \quad a^3 = 1)$

Let Δ_{ij} = Cofactor of A_{ij}

$$\therefore \Delta_{11} = a^4 - a^2 = a - a^2$$

$$\Delta_{12} = -(a^2 - a) = a - a^2$$

$$\Delta_{13} = (a - a^2)$$

$$\Delta_{21} = -(a^2 - a) = a - a^2$$

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$\Delta_{22} = a^2 - 1$ $\qquad\qquad$ $\Delta_{32} = -(a-1) = 1-a$

$\Delta_{23} = -(a-1) = 1-a$ $\qquad\qquad$ $\Delta_{33} = a^2 - 1$

$\Delta_{31} = a - a^2$

$$\therefore A^{-1} = \frac{1}{\Delta}\begin{bmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} \\ \Delta_{21} & \Delta_{22} & \Delta_{23} \\ \Delta_{31} & \Delta_{32} & \Delta_{33}\end{bmatrix}^T$$

$$= \frac{1}{3(a-a^2)}\begin{bmatrix} a-a^2 & a-a^2 & a-a^2 \\ a-a^2 & a^2-1 & 1-a \\ a-a^2 & 1-a & a^2-1\end{bmatrix}$$

$$= \frac{a-a^2}{3(a-a^2)}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \frac{a^2-1}{a-a^2} & \frac{1-a}{a-a^2} \\ 1 & \frac{1-a}{a-a^2} & \frac{a^2-1}{a-a^2}\end{bmatrix} \longrightarrow (12)$$

$$\frac{a^2-1}{a-a^2} = \frac{a^2-a^3}{a-a^2} = \frac{a(a-a^2)}{a-a^2} = a \longrightarrow (13)$$

$$\frac{1-a}{a-a^2} = \frac{a^2(1-a)}{a^2(a-a^2)} = \frac{a^2(1-a)}{a^3-a^4} = \frac{a^2(1-a)}{1-a} = a^2 \longrightarrow (14)$$

$\because a^3 = 1\ ;\ a^4 = a$

Using eqn 13 & 14 $\rightarrow$ 12

$$A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a\end{bmatrix} \longrightarrow (15)$$

Sub eqn 15 in eqn 11

$$\Rightarrow V_{sy} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a\end{bmatrix} V$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2}\end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a\end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c\end{bmatrix} \longrightarrow (16)$$

The matrix eqn 16 can be expressed as three independent linear eqns as below.

$$\therefore V_{a0} = \frac{1}{3}(V_a + V_b + V_c) \longrightarrow (17)$$

$$V_{a1} = \frac{1}{3}(V_a + aV_b + a^2V_c) \longrightarrow (18)$$

$$V_{a2} = \frac{1}{3}(V_a + a^2V_b + aV_c) \longrightarrow (19)$$

These eqns can be used to compute the sym. components of the unbalanced voltages.

Symmetrical Components of unbalanced current Vectors :-

The sym. components of unbalanced ct vectors can be obtained by an analysis similar to that of voltage vectors.

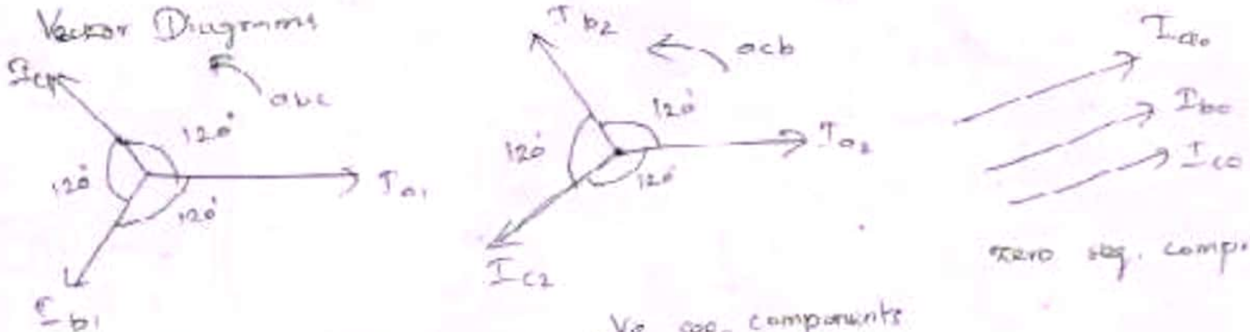
Let I_a, I_b, I_c → Unbalanced ct vectors with phase sequence abc.

I_{a1}, I_{b1}, I_{c1} → +ve sequence components of I_a, I_b & I_c resp. with phase sequence abc.

I_{a2}, I_{b2}, I_{c2} → -Ve sequence components of I_a, I_b, I_c resp. with phase sequence abc.

I_{a0}, I_{b0}, I_{c0} → Zero seq. components of I_a, I_b, I_c resp.

Vector Diagrams



+ve seq components　　-Ve seq components　　zero seq. comp.

Vector diagrams of symmetrical components of unbalanced 3-phase ct vectors

* The following eqns are used to compute the unbalanced ct vectors from knowledge of their sym. components. (Refering eqns 6, 7, 8)

$$I_a = I_{a0} + I_{a1} + I_{a2} \longrightarrow A$$

$$I_b = I_{a0} + a^2 I_{a1} + a I_{a2} \longrightarrow B$$

$$I_c = I_{a0} + a I_{a1} + a^2 I_{a2} \longrightarrow C$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \longrightarrow I$$

* The following eqns are used to compute the symmetrical components of unbalanced current vectors. (Refering eqn 17, 18, 19)

$$I_{a_0} = \frac{1}{3}\left[I_a + I_b + I_c\right] \longrightarrow D$$

$$I_{a_1} = \frac{1}{3}\left[I_a + a I_b + a^2 I_c\right] \longrightarrow E$$

$$I_{a_2} = \frac{1}{3}\left[I_a + a^2 I_b + a I_c\right] \longrightarrow F$$

$$\begin{bmatrix} I_{a_0} \\ I_{a_1} \\ I_{a_2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \longrightarrow II$$

Pbm The voltages across a 3 φ unbalanced load are $V_a = 300\angle 20°$ V, $V_b = 360\angle 90°$ V & $V_c = 500\angle -140°$ V. Phase seq. is abc. Determine the sym. components of voltages.

soln: The sym components of V_a are gn by

$$\begin{bmatrix} V_{a_0} \\ V_{a_1} \\ V_{a_2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$\therefore V_{a_0} = \frac{1}{3}(V_a + V_b + V_c)$; $V_{a_1} = \frac{1}{3}(V_a + aV_b + a^2V_c)$

$V_{a_2} = \frac{1}{3}(V_a + a^2V_b + aV_c)$

Gn that $V_a = 300\angle 20°$ V $= 281.91 + j102.61$ V

$V_b = 360\angle 90°$ V $= 0 + j360$ V

$V_c = 500\angle -140°$ V $= -383.02 - j321.39$ V

$\therefore aV_b = 1\angle 120° \times 360\angle 90° = 360\angle 210° = -311.77 - j180$ V

$a^2V_b = 1\angle 240° \times 360\angle 90° = 360\angle 330° = 311.77 - j180$ V

$aV_c = 1\angle 120° \times 500\angle -140° = 500\angle -20° = 469.85 - j171.01$ V

$a^2V_c = 1\angle 240° \times 500\angle -140° = 500\angle 100° = -86.82 + j492.4$

$V_{a_0} = \frac{1}{3}(V_a + V_b + V_c) = \frac{1}{3}(281.91 + j102.61 + j360 - 383.02 - j321.39)$

$= -33.7 + j47.07 = 57.87\angle 126°$ V

$V_{a_1} = \frac{1}{3}(V_a + aV_b + a^2V_c) = \frac{1}{3}(281.91 + j102.61 + (-311.77 - j180) - 86.82 + j492.4)$

$= -38.89 + j138.34 = 143.7\angle 106°$ V

$V_{a_2} = \frac{1}{3}(V_a + a^2V_b + aV_c) = \frac{1}{3}(281.91 + j102.61 + 311.77 - j180 + 469.85 - j171.01)$

$= 354.51 - j82.8 = 364.05\angle -13°$ V

(8)

* We know that $V_{a0} = V_{b0} = V_{c0}$

$\therefore$ The zero sequence components are

$$V_{a0} = V_{b0} = V_{c0} = 57.89\angle 120^\circ\ V$$

* W.k. that $V_{b1} = a^2 V_{a1}$; $V_{c1} = a V_{a1}$

$= 1\angle 240^\circ \times 143.7\angle 106^\circ$; $= 1\angle 120^\circ \times 143.7\angle 106^\circ$

$= 143.7\angle 346^\circ\ V$; $= 143.7\angle 226^\circ\ V$

* W.k. that $V_{b2} = a V_{a2}$; $V_{c2} = a^2 V_{a2}$

$= 1\angle 120^\circ \times 364.05\angle -13^\circ$; $= 1\angle 240^\circ \times 364.05\angle -13^\circ$

$= 364.05\angle 107^\circ\ V$; $= 364.05\angle 227^\circ\ V$

Pbm The symmetrical components of phase a voltage in a 3ϕ unbalanced s/m are $V_{a0} = 10\angle 180^\circ V$, $V_{a1} = 50\angle 0^\circ$ & $V_{a2} = 20\angle 90^\circ V$. Determine phase voltages V_a, V_b & V_c.

The phase voltages of V_a, V_b, V_c are gn by

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

$V_a = V_{a0} + V_{a1} + V_{a2}$; $V_{a0} = 10\angle 180^\circ = -10 + j0$

$V_b = V_{a0} + a^2 V_{a1} + a V_{a2}$; $V_{a1} = 50\angle 0^\circ = 50 + j0$

$V_c = V_{a0} + a V_{a1} + a^2 V_{a2}$; $V_{a2} = 20\angle 90^\circ = 0 + j20$

$a V_{a1} = 1\angle 120 \times 50\angle 0 = 50\angle 120 = -25 + j43.3$

$a^2 V_{a1} = 1\angle 240 \times 50\angle 0 = 50\angle 240 = -25 - j43.3$

$a V_{a2} = 1\angle 120 \times 20\angle 90 = 20\angle 210 = -17.32 - j10$

$a^2 V_{a2} = 1\angle 240 \times 20\angle 90 = 20\angle 330 = 17.32 - j10$

$V_a = V_{a0} + V_{a1} + V_{a2} = -10 + 50 + j0 + j20 = 40 + j20 = 44.72\angle 27^\circ V$

$V_b = V_{a0} + a^2 V_{a1} + a V_{a2} = -10 - 25 - j43.3 + (-17.32 - j10) = -52.32 - j53.3$

$= 74.69\angle -134^\circ\ V$

$V_c = V_{a0} + a V_{a1} + a^2 V_{a2} = -10 - 25 + j43.3 + 17.32 - j10$

$= -17.68 + j33.3 = 37.7\angle 118^\circ\ V$

Pbm The sym. components of phase a fault ct in a 3ϕ unbalanced s/m are $I_{a0} = 250\angle 90^\circ A$, $I_{a1} = 600\angle -90^\circ A$ & $I_{a2} = 250\angle 90^\circ A$. Determine the phase cts I_a, I_b & I_c.

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

$I_a = I_{a0} + I_{a1} + I_{a2} = 0$

$I_b = I_{a0} + a^2 I_{a1} + a I_{a2} = j350 - 519.62 + j200 - 216.51 - j125$

$= -736.13 + j525 = 904.16 \angle 145° A$

$I_c = I_{a0} + aI_{a1} + a^2 I_{a2} = j350 + 519.62 + j200 + 216.51 - j125$

$= 736.13 + j525 = 904.10 \angle 35° A$

Pbm Determine the sym. components of the unbalanced 3φ cts $I_a = 10\angle 0° A$, $I_b = 12\angle 230° A$, $I_c = 10\angle 130° A$

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$I_{a0} = \frac{1}{3}(I_a + I_b + I_c) = \frac{1}{3}(10 - 7.71 - j9.19 - 6.43 + j7.66)$

$= -1.38 - j0.51 = 1.47 \angle -160°$

$I_{a1} = \frac{1}{3}(I_a + aI_b + a^2 I_c) = \frac{1}{3}(10 + 11.82 - j2.08 + 9.85 + j1.74)$

$= 10.56 - j0.11 = 10.56 \angle -0.6° = 10.56 \angle 0°$

$I_{a2} = \frac{1}{3}(I_a + a^2 I_b + aI_c) = \frac{1}{3}(10 - 4.10 - j11.28 - 3.42 - j9.4)$

$= \frac{1}{3}(2.48 + j1.88)$

$= 0.83 + j0.063$

$= 1.04 \angle 37° \; V$

W.k. that

$I_{a0} = I_{b0} = I_{c0} = 1.47 \angle -160° A$

$I_{b1} = a^2 I_{a1} = 10.56 \angle 240° A$, $I_{c1} = a I_{a1} = 10.56 \angle 120° A$

$I_{b2} = a I_{a2} = \cancel{10.56 \angle 120°}\; 1.04 \angle 157° A$

$I_{c2} = a^2 I_{a2} = 1\angle 240° \times I_{a2} = 1.04 \angle 277° A$

$\cancel{I_{c1} = a I_{a1} = 10.56 \angle 120° A}$

Sequence Impedances & Seq. n/ws :

— are useful in the analysis of unsymmetrical faults in the power s/m.

Sequence Impedances & Sequence Networks

Sequence Impedances

The sequence impedances are impedances offered by ckt elements (or power s/m components) to positive, negative & zero sequence currents.

In any element of a ckt, the voltage drop caused by current of a certain sequence depends on the impedance of the element to that sequence ct.

+ve Sequence Impedance

The imp of a circuit element when positive sequence cts alone are following is called the +ve seq. imp.

-ve Seq Impedance

When only negative seq. currents are present, the impedance is called negative seq. imp.

Zero seq. Imp

When only zero seq. currents are present, the imp is called zero seq. imp.

The imp of any element of a balanced ckt to ~~the~~ it of one seq may be different from imp to it of another seq.

Sequence Networks :-

The single phase equivalent ckt of power s/m (impedance or reactance diagram) formed using the impedances of any one sequence only is called the sequence n/w for that particular sequence.

+ve seq. n/w : The impedance or reactance dia formed using positive sequence ~~[illegible]~~ impedance is called +ve seq n/w.

-ve seq n/w : the impedance or reactance dia formed using -ve seq. impedance is called -ve seq. n/w.

Zero Seq. n/ws :

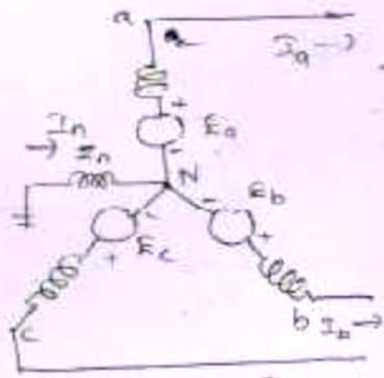
The impedance or reactance diagram formed using zero seq. impedance is called zero seq. n/w.

In unsymmetrical fault analysis of a ps, the positive, negative and zero seq. n/ws of the sln are determined & then they are inter-connected to represent the various unbalanced fault conditions.

Each seq. n/w includes the generated emfs and impedances of like seq. Also, the seq n/w carries only the ct of like seq.

Sequence Impedances & N/ws of Generator

Let consider the 3φ Equivalent ckt of a (G)



The neutral of the (G) is grounded through a reactance, Z_n.

When the (G) is delivering a balanced load or under sym. fault the neutral ct is zero.

But when the (G) is delivering an unbalanced load or during unsym. faults the neutral ct flows through Z_n.

3φ Eqvt ckt of a (G) grounded through a reactance. $I_c \rightarrow$

The (G) is designed to supply balanced 3φ Voltages.

∴ The generated emfs are of +ve seq. only.

Let E_a, E_b, E_c ⇒ Generated emf per phase in phase a, b, c resp. (positive seq emf)

Z_1 = +ve seq. imp per phase of (G)

Z_2 = −ve "

Z_{g0} = zero "

Z_n = Neutral reactance

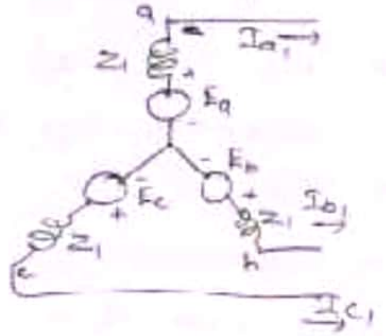
Z_0 = Total zero seq imp per phase of zero seq. n/w of (G)

→ The positive seq n/w consists of an emf in series with +ve seq imp of the G1.

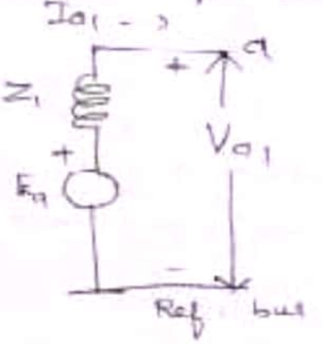
⇒ The negative & zero seq. n/w will not have any sources but include their respective seq. imp.

The positive, -ve & zero seq. ckt paths & n/ws :-

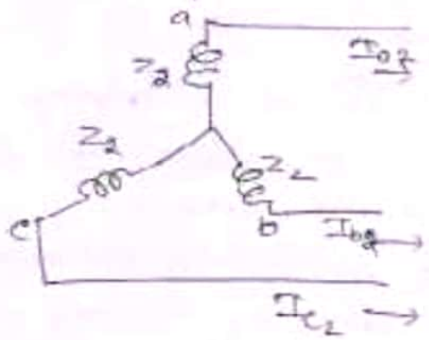
(a) +ve seq. ct paths



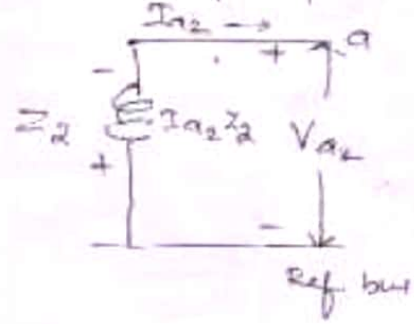
+ve seq n/w



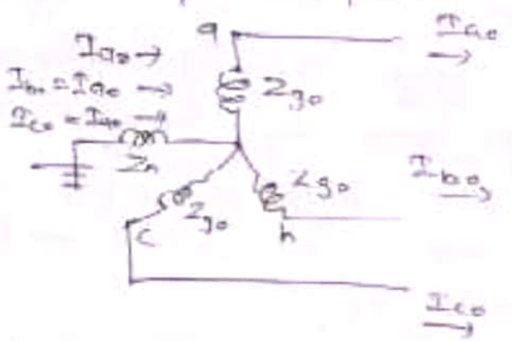
-Ve seq. ct paths



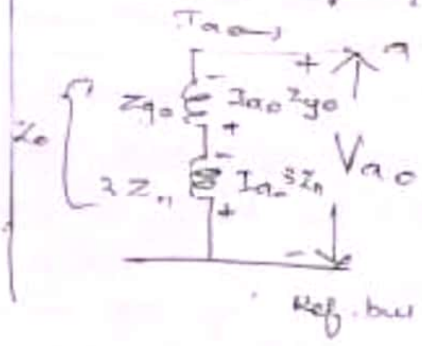
-ve seq n/w



zero seq. ct paths



zero seq n/w



The reactances in +ve seq n/w is subtransient, transient or syn. reactance depending on whether subtransient, transient or steady state conditions are being studied.

Under no load condition, the emf E_a is the induced emf per phase.

Under load or fault condition E_a is replaced by E_q' for subtransient state $E_a \Rightarrow E_g'$, for transient state $E_a \Rightarrow E_g''$.

The +ve & -ve seq cts are balanced cts and so, they will not pass through neutral reactance.

Zero seq n/w :

The ct through neutral reactance is $3I_{a0}$.

The zero seq. voltage drop from point a to gnd is $-3I_{a0}X_n - I_{a0}Z_{g0}$.

The zero seq n/w is a single phase n/w is assumed to carry only the zero seq ct of one phase, one phase. Hence the zero seq ct of one phase, must have an imp. of $3Z_n + Z_{g0}$.

∴ Total zero seq. imp per phase of a (Y) grounded through reactance $\Big]\ Z_0 = 3Z_n + Z_{g0}$

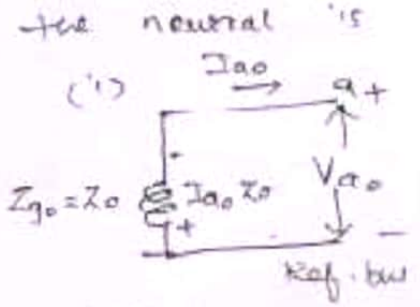
From fig, eqns for phase - a component voltages are

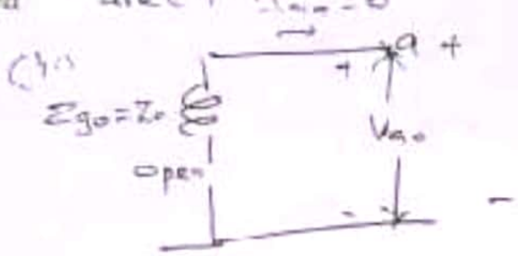
$$V_{a1} = E_a - I_{a1} Z_1$$

$$V_{a2} = -I_{a2} Z_2$$

$$V_{a0} = -I_{a0} Z_0$$

The zero seq n/w of (G) when the neutral is solidly grounded (i.e directly grounded) & when the neutral is ungrounded are (i) & (ii) resp.





In these cases there is no change in +ve & -ve seq. n/w.

The seq. n/ws of syn. (m) is same as that of generator when the directions of cts in the seq. n/w of (g) are reserved.

Sequence Imps & N/w s of Tr. line :

The imp. per phase of tr. line for balanced cts are independent of phase sequence. This is due to the transposed tr lines. Therefore, the impedance offered by the ~~symmetry o~~ tr. lines for positive & negative sequence cts are identical.

The zero seq. ct is identical (both in mag & phase) in each phase conductor & returns through the gnd, through overhead gnd wires or through both.

The gnd wires being grounded at several towers, the return cts in the ground wire may not be uniform along the entire length of tr. line.

But for +ve & -ve seq. cts there is no return ct & they have a phase diff of 120°. ∴ The magnetic field due to zero seq. ct is different from the mag. field caused by either +ve or -ve seq ct.

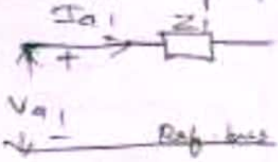
Due to difference in the magnt field, the zero sequence inductive reactance is 2 to 3.5 times the +ve seq. reactance.

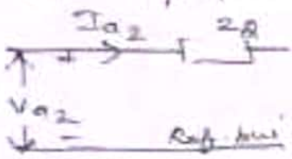
Let $Z_1 \rightarrow$ +ve seq imp of tr. line

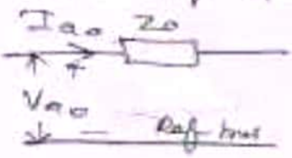
$Z_2 \rightarrow$ -ve "

$Z_0 \rightarrow$ zero "

The +ve, -ve & zero seq. impedances of the lines are represented as a series impedance on their respective sequence n/w s.

Positive seq n/w


-ve seq n/w


Zero seq n/w


Sequence Impedances & n/w s of T/f :

When the applied voltage is balanced, the +ve & -ve seq. of linear, symmetrical, static devices are identical.

∴ in a t/f the +ve & -ve seq imp are identical.

Eventhough the zero seq. imp may slightly differ from +ve & -ve seq. imp., it is normal to assume the zero seq. imp. as equal to +ve or -ve seq imp.

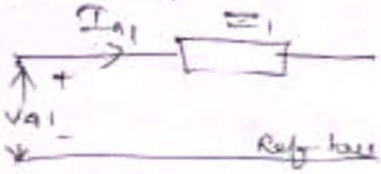
When the neutral of star connection is grounded through reactance Z_n then $3Z_n$ should be added to zero seq. imp of t/f to get the total zero impedance.

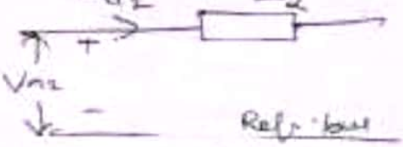
Let $Z_1 \rightarrow$ +ve seq. imp of t/f

$Z_2 \rightarrow$ -ve "

$Z_0 \rightarrow$ zero "

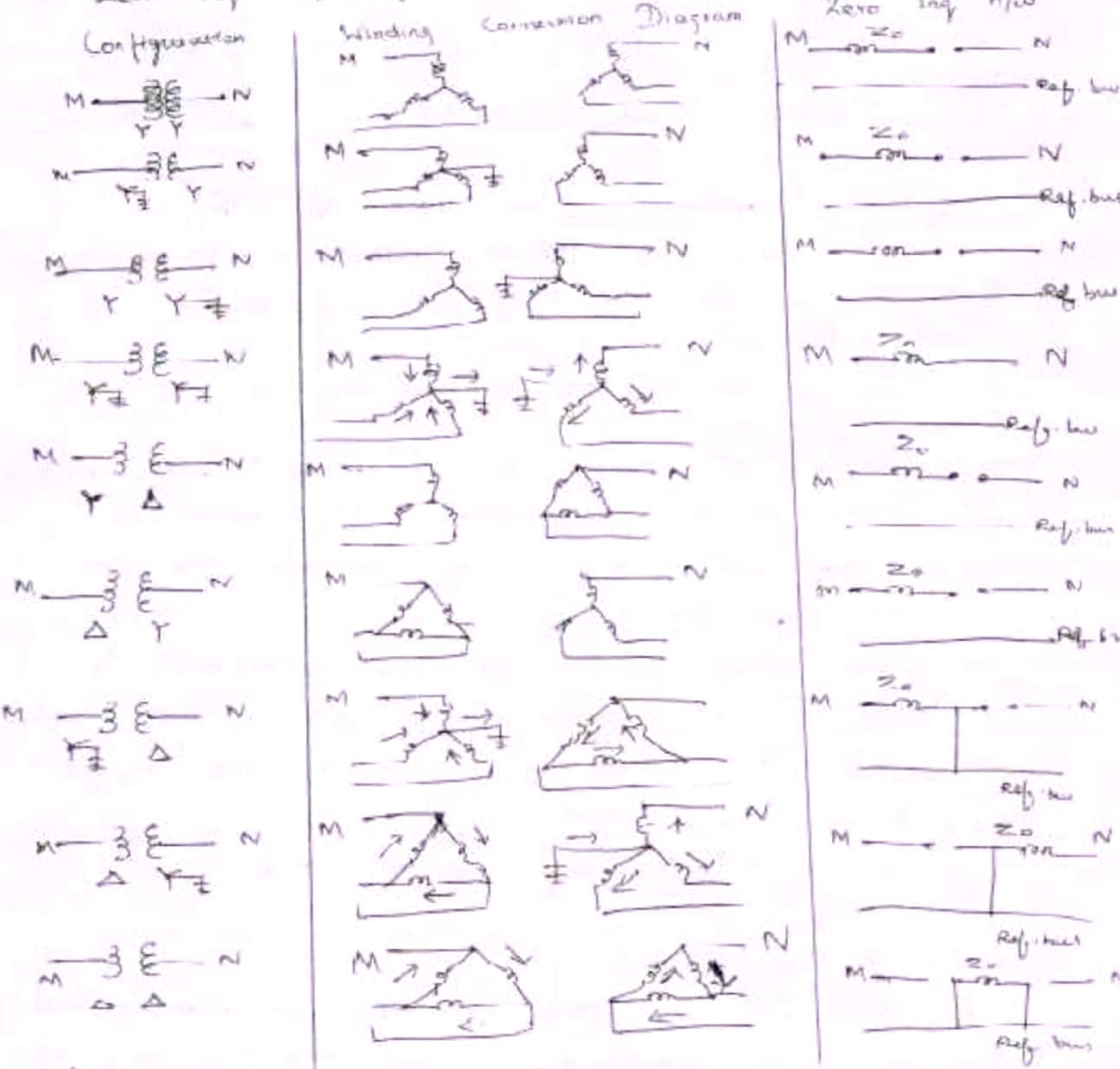
The +ve, -ve seq. imp s of t/f are represented as a series impedance in their respective seq. n/w

+ve seq n/w


-ve seq n/w


The zero seq n/w of the t/f depends on type of connections (Y or Δ) of the primary & sec wdgs of t/f & also on the grounding of neutral in Y connection.

Zero seq n/w of 3 φ t/f



The arrows on the wdgs indicate path for zero seq ct & the absence of arrows indicate that there is no path for zero seq. cts.

→ When magnetising ct is neglected the pry wdg will carry ct only if there is a ct flow on the secondary wdg ∴ The zero seq. ct can flow in the pry wdg of a t/f only if there is a path for zero seq ct in sec wdg or vice-versa.

→ If the neutral pt in the Y connected wdg is not grounded then there is no path for zero seq. ct in star connected wdg.

→ The zero seq ct flows in the star connected wdg & in the lines connected to the wdg only when the neutral pt is grounded.

→ The zero seq ct can circulate in the delta connected wdg but the zero seq. ct can't flow through the lines connected to the delta connected wdg.

Sequence Imps & N/ws of Loads

In balanced Y or Δ connected loads the positive, negative & zero seq. impedances are equal.

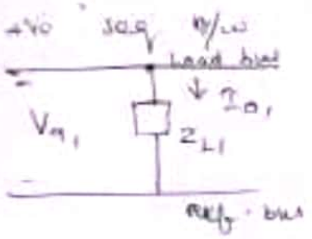
When the neutral pt of star connected load is grounded through a reactance Z_n then $3Z_n$ is added to the zero seq imp. to get the total zero seq. imp of load.

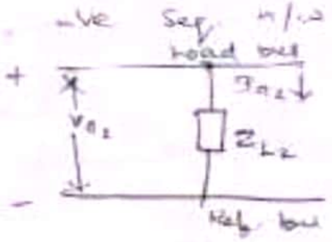
Let Z_{L1} → +ve seq imp of load

Z_{L2} → -ve "

Z_{L0} → zero "

+ve & -ve seq imp. of load are represented as a shunt imp. in their respective seq n/w.

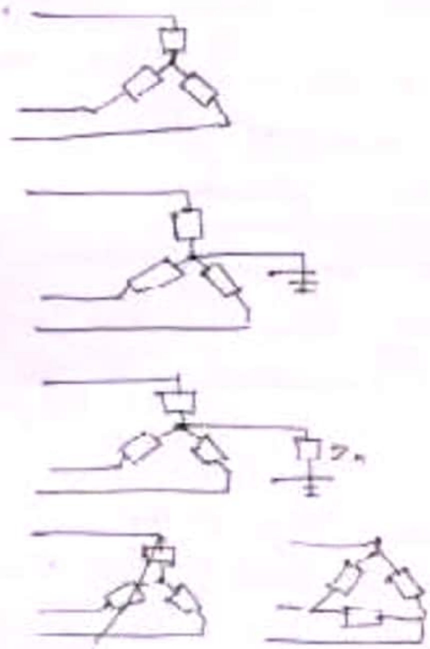




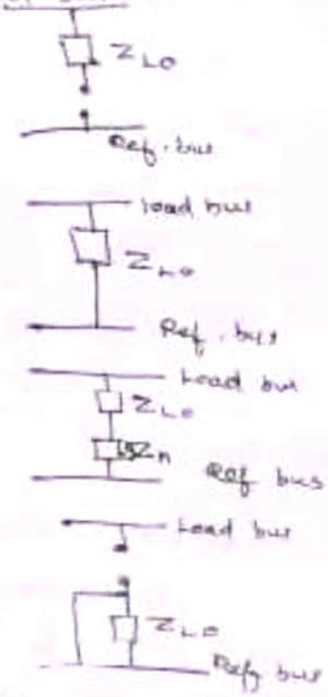
The zero seq. n/w of the 3-ϕ load depends on the type of connection, i.e Y or Δ connection. The zero seq ct will flow in n/w only if a return path exists for it.

Zero seq n/ws of loads

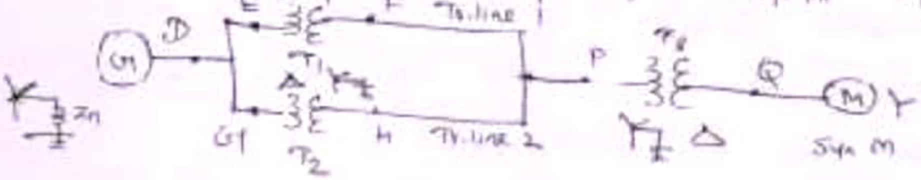
Connection diagram



Zero seq n/w



Pbm Draw the +ve, -ve, zero seq n/w diagram of the P.S.



Soln Let $X_{G1,1}$ → +ve seq Reactance of Gen. G

$X_{M,1}$ → " Motor M

$X_{T1,1}$ → " T/f T_1

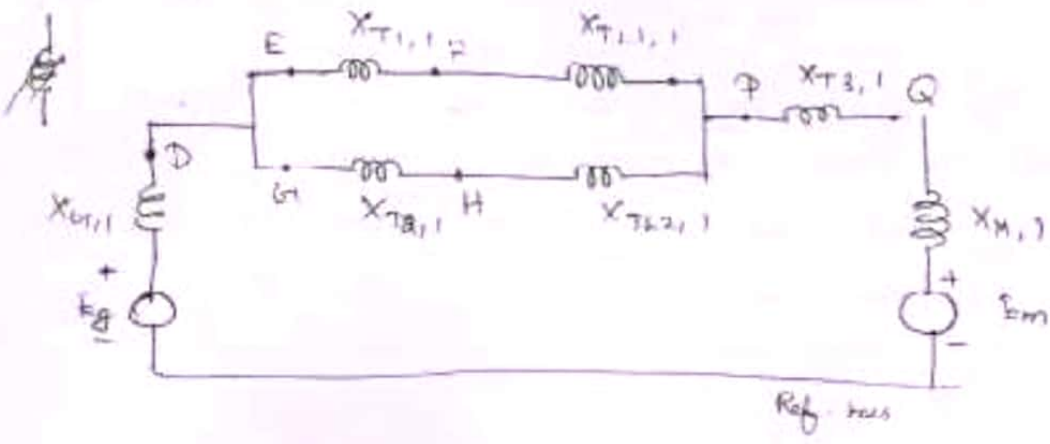
$X_{T2,1}$ → " T/f T_2

$X_{T3,1}$ → " T/f T_3

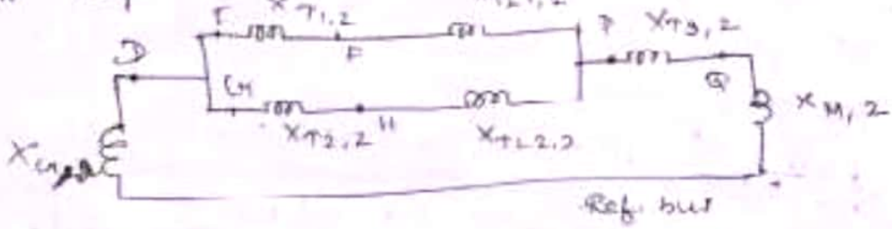
$X_{TL1,1}$ → " Tr. line 1

$X_{TL2,1}$ → " Tr. line 2

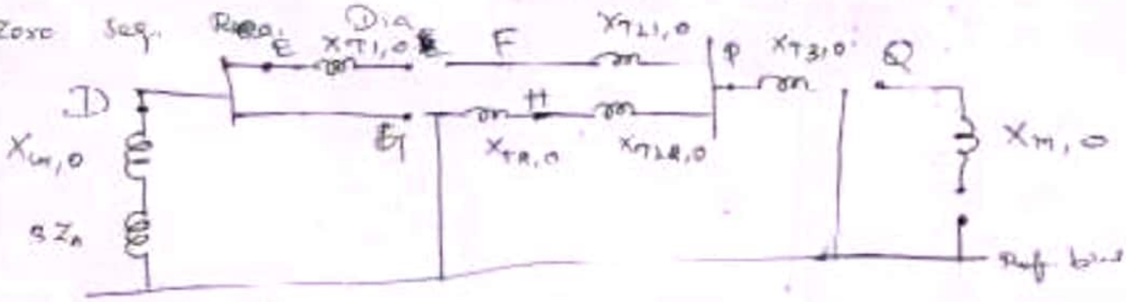
Positive seq. Reactance Diagram



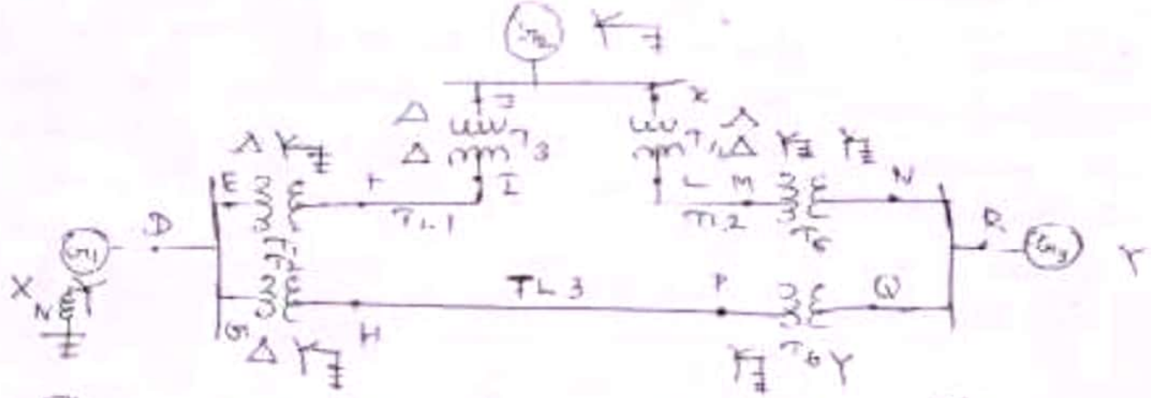
-ve Seq. Rea. Dia



Zero Seq. Rea. Dia

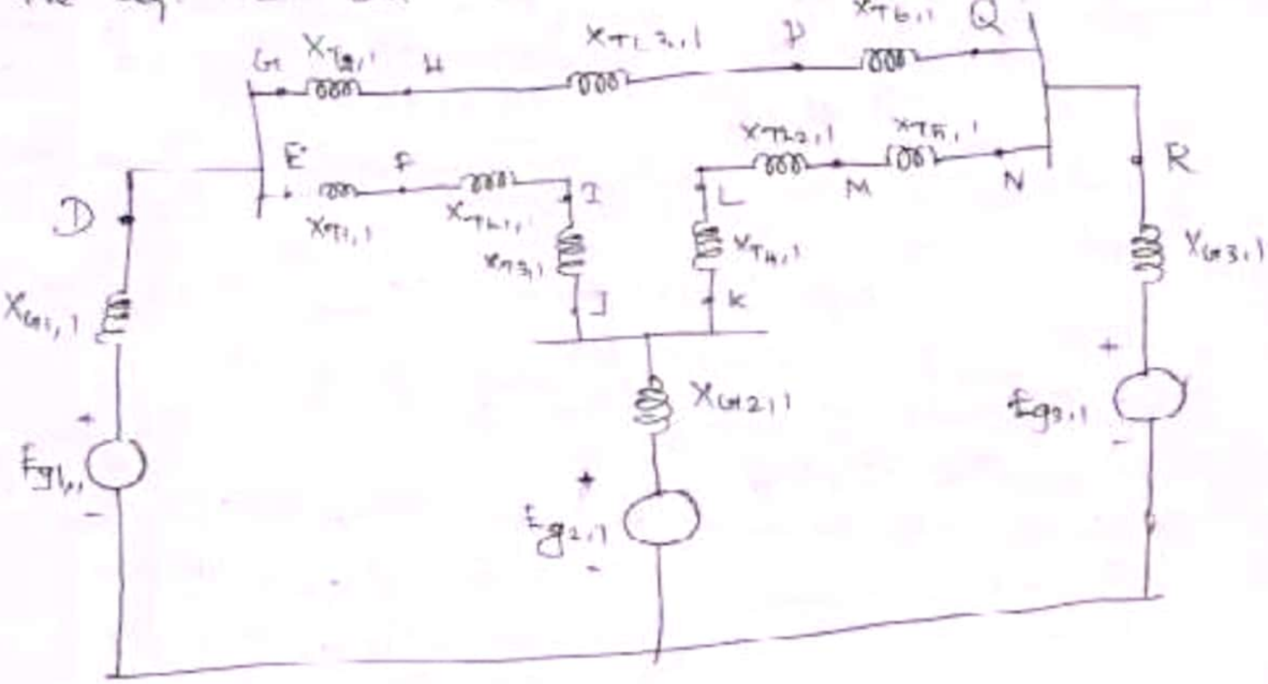


Pbm



Draw +ve, -ve & zero seq. Rea. Dia

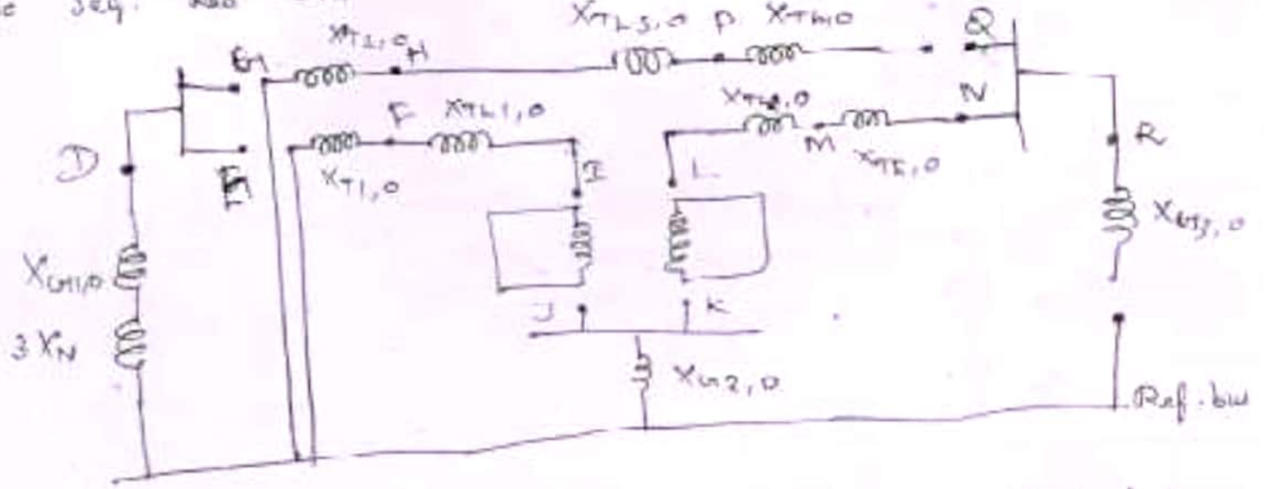
+ve Seq. Rea. Dia



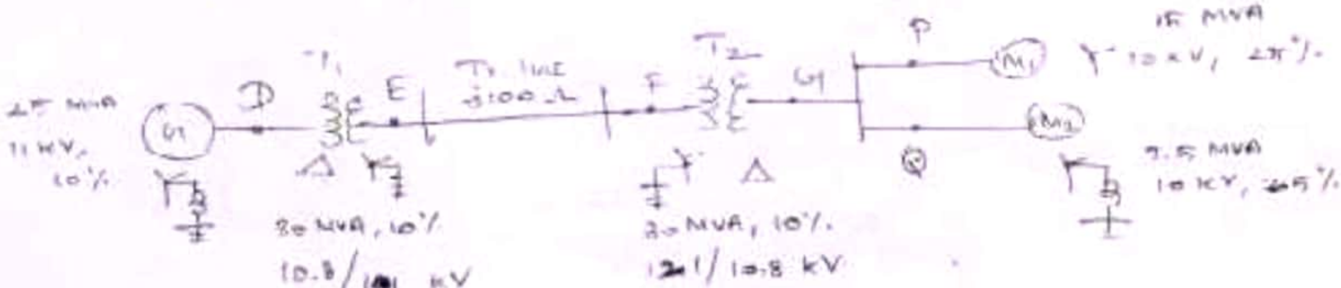
-ve Seq. Rea. Dia

As positive seq. Dia w/o source (G, M)

Zero seq. Rea Dia



Pbm X. Determine the positive, negative & zero seq n/ws for the s/m shown in fig. Assume zero seq. reactances for the generator & syn motors as 0.06 p.u. Current limiting reactors of 2.5 Ω are connected in the neutral of the (G) & (M) No 2. The zero seq rea. of the tr. line is j300 Ω.



Soln Let us choose the (G) ratings as new base values for entire s/m.

Base Megavolt ampere $MVA_{base,new} = 25$ MVA

Base kilovolt $kV_{b,new} = 11$ kV.

Seq. Reactance of Gen G :-

Does not change.

Positive seq reactance of Gen, $X_{G1} = 10\% = \frac{10}{100} = 0.1$ p.u

Negative " $X_{G,2} = 0.1$ p.u

Zero " $X_{G,0} = 0.06$ p.u

p.u Value of generator neutral reactance } $X_{GN} = \frac{\text{Actual Neutral Reactance}}{\text{Base impedance}}$

Base imp, $Z_b = \frac{(kV_{b,new})^2}{MVA_{b,new}} = \frac{11^2}{25} = 4.84\ \Omega$

$\therefore X_{n,N} = \frac{2.5}{4.84} = 0.517$ p.u

Seq. Reactance of T/f T_1 :-

$$\left.\begin{array}{l}\text{New p.u reactance}\\ \text{of T/f } T_1\end{array}\right\} = X_{p.u\,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \frac{MVA_{b,new}}{MVA_{b,old}}$$

Here $X_{pu,old} = 10\% = 0.1$ p.u $kV_{b,new} = 11$ kV

$kV_{b,old} = 10.8$ kV $MVA_{b,new} = 25$ MVA

$MVA_{b,old} = 30$ MVA

$\therefore$ New p.u Reactance of T/f $T_1 = 0.1 \times \left(\frac{10.8}{11}\right)^2 \times \left(\frac{25}{30}\right)$

$= 0.08$ p.u

$\Rightarrow$ In t/f the specified reactance is +ve seq. reactance. Also we assume that the +ve, -ve & zero seq. reactance of the t/f are equal.

$\therefore$ +ve seq. Rea. of T/f T_1, $X_{T1,1} = 0.08$ p.u

-ve " $X_{T1,2} = 0.08$ p.u

zero " $X_{T1,0} = 0.08$ p.u

Seq. Rea. of Tr. line :

$$\left.\begin{array}{l}\text{The base kv on HT side}\\ \text{of t/f } T_1\end{array}\right\} = \text{Base kv on LT side} \times \frac{\text{HT V Rating}}{\text{LT V Rating}}$$

$= 11 \times \frac{121}{10.8} = 123.24$ kV

Now $kV_{b,new} = 123.24$ kV

Base impedance $= \frac{(kV_{b,new})^2}{MVA_{b,new}} = \frac{123.24^2}{30} = 506.27\ \Omega$

p.u Reactance of Tr. line $= \frac{\text{Actual Reactance}}{\text{Base impedance}} = \frac{100}{506.27} = 0.178$ p.u

$\Rightarrow$ The specified reactance in single line diagram is +ve seq. reactance. Also the negative sequence reactance of a tr. line is same as that of +ve seq. reactance.

$\therefore$ +ve seq. reactance of tr. line, $X_{TL1,1} = 0.198$ p.u

-ve " $X_{TL1,2} = 0.198$ p.u

p.u value of zero sequ. rea. of tr.line $X_{TL1,0} = \frac{\text{zero seq. rea}}{\text{Base imp}}$

$= \frac{300}{50.577} = 0.593$ p.u

Seq. Reactance of T/f T_2 :

The ratings & connections of T/f T_1 & T_2 are identical & so the seq. reactances of T_1 & T_2 are same.

+ve seq. rea. of T/f T_2, $X_{T2,1} = 0.08$ p.u

-ve " $X_{T2,2} = 0.08$ p.u

zero " $X_{T2,0} = 0.08$ p.u

Seq. Rea. of Syn (M) M_1

Base kV on LT side of T/f T_2 } = Base kV on HT side $\times \frac{\text{LT V. Rating}}{\text{HT V. Rating}}$

$= 123.24 \times \frac{10.8}{121} = 11$ kV

$\therefore$ $kV_{b,new} = 11$ kV

New p.u Rea. of M, M_1 = $X_{p.u\,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \frac{MVA_{b,new}}{MVA_{b,old}}$

$= 0.25 \times \left(\frac{10}{11}\right)^2 \times \left(\frac{25}{15}\right)$

$= 0.344$ p.u

$\Rightarrow$ The rea. specified in single line dia is +ve seq. reactance. Also the -ve seq rea. of syn. (M) is same as that of +ve seq. rea.

$\therefore$ +ve seq. Reactance of M, M_1, $X_{M1,1} = 0.344$ p.u

-ve " M_1 $X_{M1,2} = 0.344$ p.u

Zero " $X_{M1,0} = 0.082$ p.u

zero seq. rea of (M), M_1 on new bases } $X_{M1,0} = X_{p.u\,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \frac{MVA_{b,new}}{MVA_{b,old}}$

$= 0.06 \times \left(\frac{10}{11}\right)^2 \times \left(\frac{25}{15}\right)$

$= 0.083$ p.u

Seq. Rea. of Syn (M) M_2 :

New p.u Rea. of (M) M_2 , $= X_{p.u\,old} \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \frac{MVA_{b,new}}{MVA_{b,old}}$

Here $X_1 = 0.25 \times \left(\frac{10}{11}\right)^2 \times \left(\frac{25}{7.5}\right) = 0.689$ p.u

+ve seq. Rea of (M) M_2, $X_{M2,1} = 0.689$ p.u

-ve " $X_{M2,2} = 0.689$ p.u

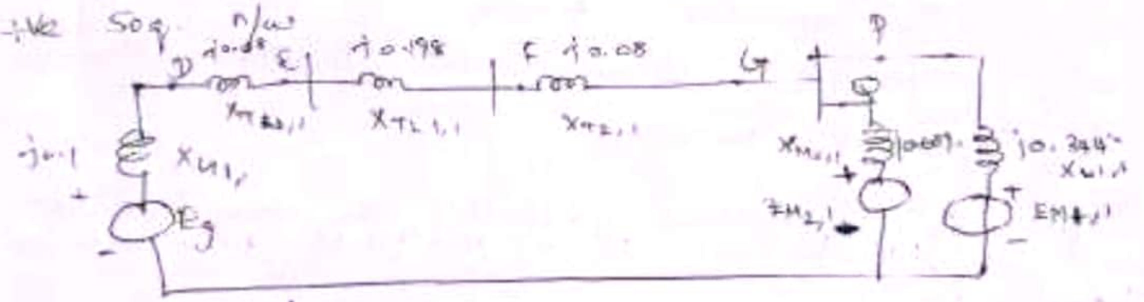
The zero seq reactance of (M) M_1 on the new bases, $X_{M2,0} = X_{p.u\,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}}\right)^2 \times \frac{MVA_{b,new}}{MVA_{b,old}}$

$= 0.06 \times \left(\frac{10}{11}\right)^2 \times \left(\frac{25}{7.5}\right) = 0.165$ p.u

p.u value of (M) neutral Reactance $X_{MN} = \frac{\text{Actual neutral Rea}}{\text{Base imp}}$

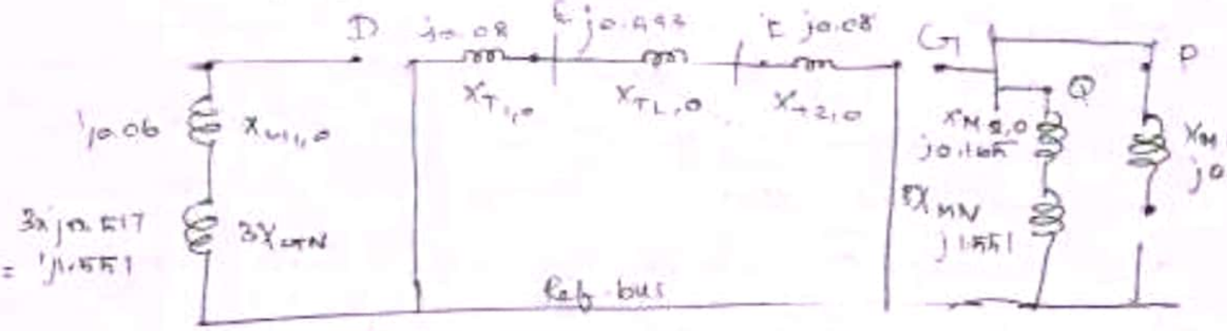
$= \frac{2.5}{4.84} = 0.511$ p.u

+ve Seq n/w



-ve Seq n/w
Same as +ve seq n/w (w/o sources E_g, $E_{M1,1}$, $E_{M2,1}$)

Zero seq n/w



Unsymmetrical Fault Analysis

When the n/w is unsymmetrically faulted or loaded, neither the phase ckt nor the phase voltages will possess three phase symmetry.

Most of the s/m faults are unsymmetrical faults. It consists of unsym. short ckt faults or unsym. faults through impedance, or open conductor faults. If the insulation of the s/m fails at any pt or if a conducting object comes in contact with a bare conductor, an unsym. short ckt fault is said to occur.

If unsym. fault occurs, the unbalanced ckt will flow in the s/m. We are using sym. components to analyze unsym. faults.

To determine positive & negative and zero seq. impedance, we can use Thevenin's theorem or Bus impedance matrix.

Types of unsymmetrical faults: The unsy. faults are the faults in which the fault cts in the three phases are unequal.

Types:
* Line to Ground fault (L-G)
* Line to line fault (L-L)
* Double line to Ground fault (L-L-G)
* One or two Open conductor faults.

Causes :- Lightning; wind damage, tree falling across lines, vehicles colliding with towers or poles, birds shorting lines, breaks due to excessive ice loading or snow loading, salt spray.

Short ckt analysis of unbalanced low order systems :-

1. Draw the +ve, -ve & zero seq. n/ws with their appropriate description.

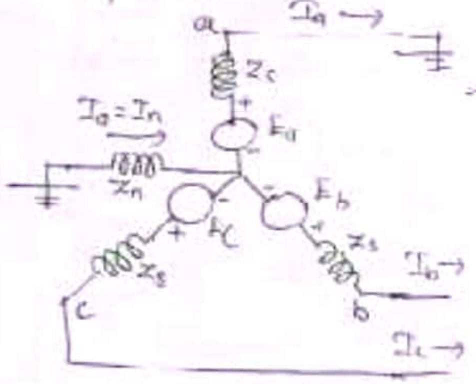
2. Choice of type of fault (L-G, L-L or L-L-G) & location of fault & mathematical description for the particular type of fault.

3. Using Thevenin's theorem on bus impedance matrix determine the soln of the n/w eqns.

Fault ct, post fault ckt, post fault voltages, are found at the point of fault, all the bus voltages & the line flows.

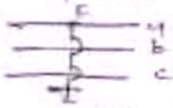
Single Line to Ground fault on an unloaded Generator

The most common type fault, single line to grd fault is caused by lightning or by conductors making contact with grounded structures.



The ckt dia shows a single line to grd fault on an unloaded Y (star) connected Gr with its neutral grounded through a reactance Z_n.

Assuming Fault occurs on phase 'a'.



Now the fault ct, $I_f = I_a$

Since the generator is unloaded the ct in other phases are zero.

The condition at the fault is expressed by the following eqns

$$I_b = 0 \; ; \quad I_c = 0 \; ; \quad V_a = 0 \; ;$$

The sym. components of the currents are gn by

$$\begin{bmatrix} I_{a_0} \\ I_{a_1} \\ I_{a_2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \longrightarrow 1$$

On substituting the conditions $I_b = I_c = 0$ in the sym. components of ckt we get

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix}$$

$$I_{a0} = \frac{1}{3} I_a \; ; \quad I_{a1} = \frac{1}{3} I_a \; ; \quad I_{a2} = \frac{1}{3} I_a$$

$$\therefore \boxed{I_{a0} = I_{a1} = I_{a2} = I_a/3} \rightarrow 2$$

From the seq. n/w's of the G we get the following matrix eqn

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \cancel{\frac{1}{3}}\begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

On sub $I_{a0} = I_{a1}$ & $I_{a2} = I_{a1}$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}\begin{bmatrix} I_{a1} \\ I_{a1} \\ I_{a1} \end{bmatrix}$$

$$V_{a0} = - Z_0 I_{a1} \longrightarrow A$$

$$V_{a1} = E_a - Z_1 I_{a1} \longrightarrow B$$

$$V_{a2} = - Z_2 I_{a1} \longrightarrow C$$

On adding eqns A, B, C

$$V_{a0} + V_{a1} + V_{a2} = -I_{a1} Z_0 + E_a - Z_1 I_{a1} - Z_2 I_{a1} \longrightarrow D$$

Single phase a is shorted to gnd, & so eqn D can be

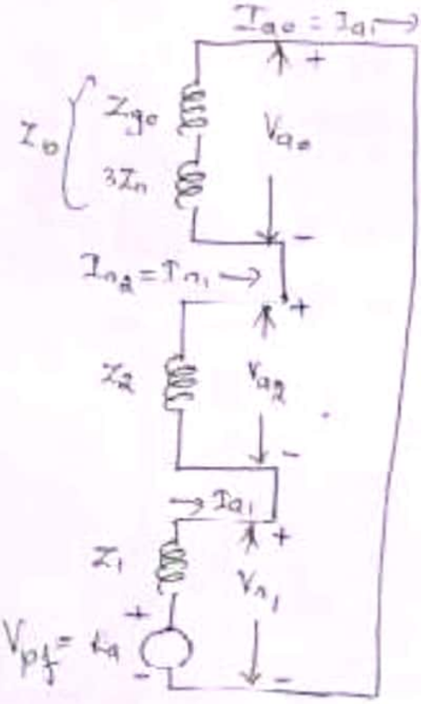
$$V_a = V_{a0} + V_{a1} + V_{a2} = 0$$

written as

$$-I_{a1} Z_0 + E_a - I_{a1} Z_1 - I_{a1} Z_2 = 0$$

$$\boxed{I_{a1} = \frac{E_a}{Z_0 + Z_1 + Z_2}}$$

Using these eqn the eqnt ckt of G during single line to ground fault is drawn as shown in fig.



Connection of sequence n/w of an unloaded (G) for Line to ground fault on phase - a.

Here the +ve, -ve & zero seq n/ws of the (G) are connected in series.

If the neutral of the (G) is not grounded, the zero seq n/w is open - circuited & Z_0 is infinite.

Under this condition I_{a1} is zero & so I_{a2} & I_{a0} must be zero. ∵ no path exists for the flow of ct in the fault unless the (g) neutral is grounded.

Here the fault ct is gn by

$$\boxed{I_f = I_a = 3\, I_{a1}}$$

Seqn cts are gn by

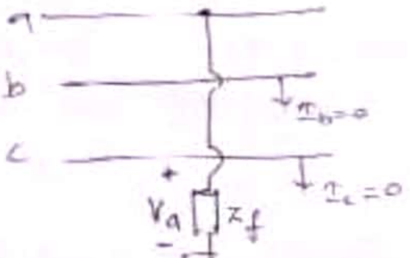
$$\boxed{I_{a1} = I_{a2} = I_{a0} = I_a/3}$$

$$\boxed{I_{a1} = \frac{V_{pf}}{Z_1 + Z_2 + Z_0}}$$

Single line to ground fault through an impedance

There may be situations in which the fault path includes an impedance b/w the faulted points. In these situations, the fault impedance are included at appropriate points in the ckts obtained by connecting seq. n/ws.

Let a single line to ground fault at point F in a PS through a fault impedance Z_f can be represented by connecting two stubs as shown in fig.



At that pt F, the following relations exist

$$\left.\begin{array}{l} I_b = 0 \;,\; I_c = 0 \\ V_a = Z_f I_a \end{array}\right\} \rightarrow (1)$$

The sym. components of the fault ct are

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore I_{a1} = I_{a2} = I_{a0} = \frac{I_a}{3} \longrightarrow (2)$$

$$\text{or} \quad I_a = 3 I_{a1} \longrightarrow (3)$$

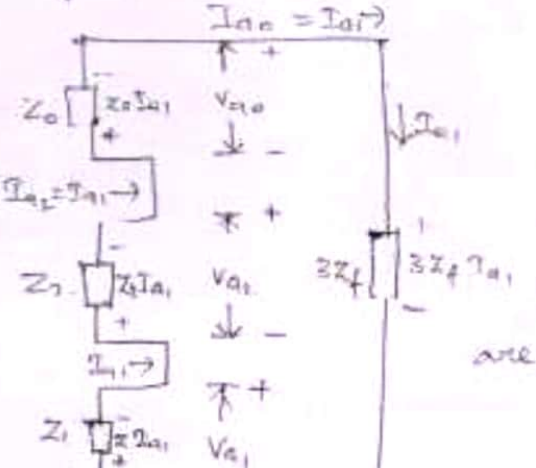
Expressing the voltage V_a in terms of sym. components we have $V_a = V_{a1} + V_{a2} + V_{a0} \longrightarrow (4)$

Use eqn (1) & (3)

$$\Rightarrow V_a = Z_f I_a = 3 Z_f I_{a1} = V_{a1} + V_{a2} + V_{a0} \longrightarrow (5)$$

From eqn 2 & 5, all sequence cts are equal & the sum of sequence voltages equals $3 Z_f I_{a1}$.

Connection of seq n/w for a single line to gnd fault through an imp. Z_f



From fig,

$$I_{a1} = \frac{V_{pf}}{(Z_1 + Z_2 + Z_0) + 3 Z_f} \longrightarrow (6)$$

Fault ct $I_f = I_a = 3 I_{a1}$

$$= \frac{3 V_{pf}}{(Z_1 + Z_2 + Z_0) + 3 Z_f} \longrightarrow (7)$$

Sym. components of phase a voltages are calculated by writing KVL

$$V_{a1} = V_{pf} - Z_1 I_{a1}$$

$$V_{a2} = -Z_2 I_{a1}$$

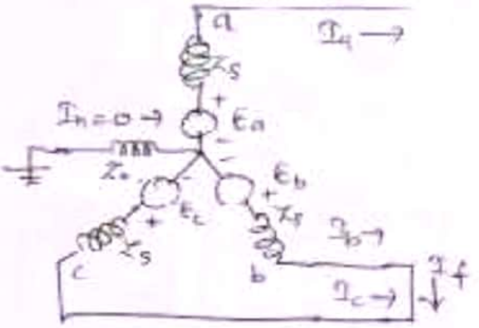
$$V_{a0} = -Z_0 I_{a1}$$

Phase voltages is given by

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

Line to Line Fault of an unloaded (G)

Let consider phase b & phase c are shorted.
The ckt dia shows a line to line fault of an unloaded ckt.



Line to line fault

Now the fault ckt $I_f = I_b = -I_c$
Since the (G) is unloaded, the ckt in phase a is zero.

The conditions at the fault are expressed by the following eqns.

$$\left.\begin{aligned} V_b = V_c \quad &; \quad I_a = 0 \\ I_b + I_c = 0 \quad &\Rightarrow \quad I_b = -I_c \end{aligned}\right\} \rightarrow 1$$

with $V_c = V_b$, the sym. components of voltages are gn by

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_b \end{bmatrix} \rightarrow 2$$

$$V_{a0} = \frac{1}{3}(V_a + 2V_b)$$

$$\left.\begin{aligned} V_{a1} &= \frac{1}{3}(V_a + aV_b + a^2 V_b) \\ V_{a2} &= \frac{1}{3}(V_a + a^2 V_b + aV_b) \end{aligned}\right\} \begin{aligned} &\rightarrow 3 \\ &\rightarrow 4 \end{aligned}$$

From eqns 3 & 4 $\Rightarrow$ $V_{a1} = V_{a2} \rightarrow 5$

The sym. components of ckt are gn by

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

On sub $I_a = 0$; $I_b = -I_c$

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} 0 \\ -I_c \\ I_c \end{bmatrix}$$

$$I_{a0} = \frac{1}{3}(-I_c + I_c) = 0$$

$$I_{a1} = \frac{1}{3}(-aI_c + a^2 I_c) \rightarrow 6$$

$$I_{a2} = \frac{1}{3}(-a^2 I_c + aI_c) \rightarrow 7$$

~~From eqn 6 & 7~~

$$I_{a2} = \frac{1}{3}\left(-a^2 I_c + a I_c\right) = -\frac{1}{3}\left(-a I_c + a^2 I_c\right) \rightarrow 7$$

from eqns 6 & 7

$$I_{a2} = -I_{a1} \rightarrow (8)$$

From the seq n/w's of the (G) we get the following matrix eqns.

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

On sub $I_{a0} = 0$; $I_{a2} = -I_{a1}$

$$\Rightarrow \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} 0 \\ I_{a1} \\ -I_{a1} \end{bmatrix}$$

$$V_{a0} = 0$$

$$V_{a1} = E_a - I_{a1} Z_1 \longrightarrow 9$$

$$V_{a2} = Z_2 I_{a1} \longrightarrow 10$$

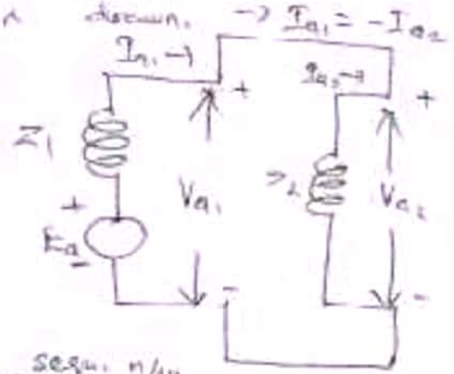
From eqn 5 we know $V_{a1} = V_{a2}$

$$\therefore E_a - I_{a1} Z_1 = Z_2 I_{a1}$$

$$I_{a1}(Z_1 + Z_2) = E_a$$

$$\therefore \boxed{I_{a1} = \frac{E_a}{Z_1 + Z_2}}$$

Using this eqn the eqnt ckt of (G) during line to line fault is drawn. $\rightarrow I_{a1} = -I_{a2}$



seq. n/w (line to line fault b/w phase b & c)

since $V_{a1} = V_{a2}$, here the +ve & -ve seq. n/w's of the (G) must be in ||el.

since $V_{a0} = 0$, the zero seq. n/w is shorted & so it need not be considered.

Since this type of fault does not involve ground, the neutral ct $I_n = 0$.

Hence the presence or absence of grounded neutral at the Gr doesn't affect line fault, ct.

Here fault ct, $\boxed{I_f = I_b = -I_c}$.

The unbalanced ct I_a, I_b & I_c are related to the sym. comp of cts are

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

$$I_b = I_{a0} + a^2 I_{a1} + a I_{a2}$$

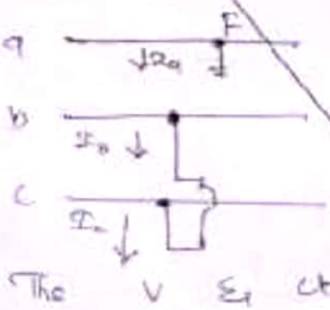
On sub $I_{a0} = 0$ & $I_{a2} = -I_{a1}$

$$\Rightarrow I_b = a^2 I_{a1} - a I_{a1}$$

$$= I_{a1}(a^2 - a)$$

$$\therefore \boxed{I_f = I_b = I_{a1}(a^2 - a)}$$

Line to line fault through impedance:



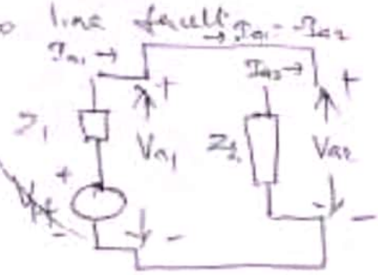
The following relations exist at the fault

$V_b = V_c$; $I_a = 0$ $I_b = -I_c$

The V & cts under this fault condition are same as that of line to line fault on an unloaded Gr.

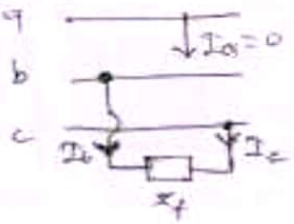
Hence for a line to line fault

$$\boxed{\begin{aligned} V_{a1} &= V_{a2} \\ I_{a1} &= \frac{V_{pf}}{Z_1 + Z_2} \end{aligned}}$$



Fault through Impedance in line to line fault

A line to line fault at pt F in a ps the phases b & c through a fault imp Z_f can be represented by connecting three stubs as show fig



The cts & V_s at the fault can be expressed as

$$I_a = 0 \;;\; I_b = -I_c \longrightarrow 1$$

Also $V_b - V_c = I_b Z_f$

$$V_c = V_b - I_b Z_f \longrightarrow 2$$

The sym comp of the cts after sub $I_a = 0$ & $I_c = -I_b$

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix}$$

$$I_{a0} = \frac{1}{3}(I_b - I_b) = 0$$

$$I_{a1} = \frac{1}{3}(aI_b - a^2 I_b)$$

$$I_{a2} = \frac{1}{3}(a^2 I_b - aI_b) = -\frac{1}{3}(aI_b - a^2 I_b)$$

$$= -I_{a1}$$

$$\therefore I_{a0} = 0 \;;\; I_{a2} = -I_{a1} \longrightarrow 3$$

The line cts can be obtained from the matrix eqn after sub these conditions (5)

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\begin{bmatrix} 0 \\ I_{a1} \\ -I_{a1} \end{bmatrix}$$

$$I_b = (a^2 I_{a1} - aI_{a1}) = I_{a1}(a^2 - a) \longrightarrow 4$$

$$= I_{a1}\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2} + \frac{1}{2} - j\frac{\sqrt{3}}{2}\right) = -j\sqrt{3}\, I_{a1}$$

(or)

$$= I_{a1}(-0.5 - j0.866 - (-0.5 + j0.866))$$

$$= I_{a1}(-0.5 - j0.866 + 0.5 - j0.866)$$

$$= I_{a1}(-1.732j)$$

The sym. components of phase voltages are

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \cdot \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

On sub $V_c = V_b - I_b Z_f$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_b - I_b Z_f \end{bmatrix}$$

$$V_{a1} = \frac{1}{3}\left(V_a + aV_b + a^2 V_b - a^2 I_b Z_f \right)$$

$$3 V_{a1} = V_a + V_b(a + a^2) - a^2 I_b Z_f \longrightarrow (5)$$

$$V_{a2} = \frac{1}{3}\left(V_a + a^2 V_b + a V_b - a I_b Z_f \right)$$

$$3 V_{a2} = V_a + V_b(a^2 + a) - a Z_f I_b \longrightarrow (6)$$

On subtracting eqn 6 from 5

$$3V_{a1} - 3V_{a2} = \cancel{V_a} + V_b(a + a^2) - a^2 I_b Z_f - \cancel{V_a}$$
$$- V_b(a + a^2) + a Z_f I_b$$
$$= (-a^2 + a) Z_f I_b$$

On -1

$$= \left(\frac{1}{2} + j\frac{\sqrt{3}}{2} - \frac{1}{2} + j\frac{\sqrt{3}}{2} \right) Z_f I_b$$

$$= j\sqrt{3}\, I_b Z_f$$

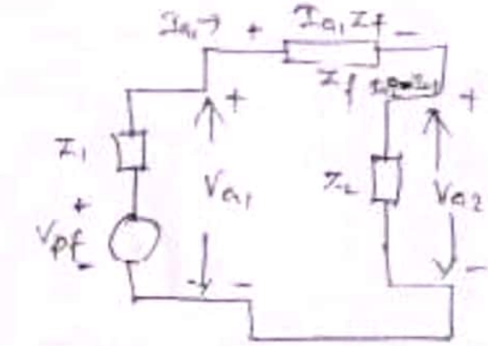
$$\therefore V_{a1} - V_{a2} = j\frac{\sqrt{3}}{3} I_b Z_f = j\frac{1}{\sqrt{3}} I_b Z_f \rightarrow (7)$$

On sub for I_b from eqn 4 in 7

$$V_{a1} - V_{a2} = j\frac{1}{\sqrt{3}} Z_f \left(-j\sqrt{3}\, I_{a1} \right)$$

$$= Z_f I_{a1} \longrightarrow (8)$$

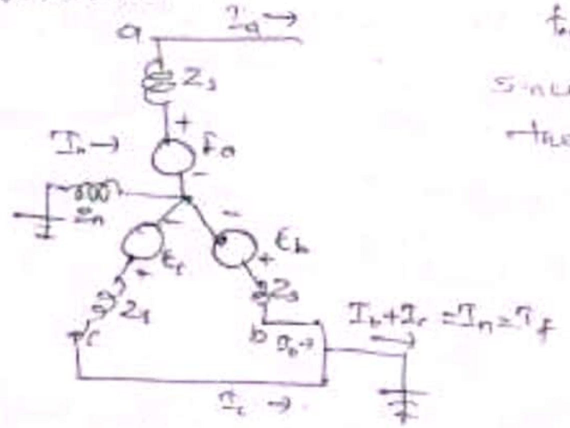
The eqns 7 & 8 suggest ||el connection of +ve & -ve seq. n/ws through a series impedance Z_f.



$$I_{a_1} = \frac{V_{pf}}{Z_1 + Z_2 + Z_f}$$

$$I_f = I_b = -j\sqrt{3}\, I_{a_1}$$

Double line to ground fault on an unloaded Generator :



fault at $I_f = I_b + I_c$
Since the G is unloaded, the ct in phase a is zero.

The conditions at the fault are expressed by the following eqns

$$V_b = 0 \quad ; \quad V_c = 0 \quad ; \quad I_a = 0$$

With $V_b = V_c = 0$, the sym. comp of Vs are

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ 0 \\ 0 \end{bmatrix}$$

$$V_{a0} = V_{a1} = V_{a2} = V_a/3 \longrightarrow 1$$

From the seq n/ws of the Gen,

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \rightarrow A$$

$$V_{a1} = E_a - Z_1 I_{a1} \rightarrow 2$$

From eqn 1

$V_{a0} = V_{a1} = V_{a2}$ & from eqn 2

$$V_{a1} = E_a - I_{a1} Z_1$$

On sub these in A

$$\begin{bmatrix} E_a - I_{a_1} Z_1 \\ E_a - I_{a_1} Z_1 \\ E_a - I_{a_1} Z_1 \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_{a_0} \\ I_{a_1} \\ I_{a_2} \end{bmatrix}$$

On rearranging this eqn

$$\begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix} \begin{bmatrix} I_{a_0} \\ I_{a_1} \\ I_{a_2} \end{bmatrix} = \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} E_a - I_{a_1} Z_1 \\ E_a - I_{a_1} Z_1 \\ E_a - I_{a_1} Z_1 \end{bmatrix} \longrightarrow (B)$$

Let $Z = \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{bmatrix}$ $\therefore Z^{-1} = \begin{bmatrix} 1/Z_0 & 0 & 0 \\ 0 & 1/Z_1 & 0 \\ 0 & 0 & 1/Z_2 \end{bmatrix}$

To find Z^{-1}

$$Z^{-1} = \frac{\text{Adjoint of } Z}{\text{Determinant of } Z} = \frac{Z^T_{cof}}{\Delta Z}$$

$$\Delta Z = \begin{vmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{vmatrix} = Z_0 Z_1 Z_2$$

$$Z^T_{cof} = \begin{bmatrix} Z_1 Z_2 & 0 & 0 \\ 0 & Z_0 Z_2 & 0 \\ 0 & 0 & Z_0 Z_1 \end{bmatrix}^T = \begin{bmatrix} Z_1 Z_2 & 0 & 0 \\ 0 & Z_0 Z_2 & 0 \\ 0 & 0 & Z_0 Z_1 \end{bmatrix}$$

$$\therefore Z^{-1} = \frac{1}{Z_0 Z_1 Z_2} \begin{bmatrix} Z_1 Z_2 & 0 & 0 \\ 0 & Z_0 Z_1 & 0 \\ 0 & 0 & Z_0 Z_1 \end{bmatrix} = \begin{bmatrix} 1/Z_0 & 0 & 0 \\ 0 & 1/Z_1 & 0 \\ 0 & 0 & 1/Z_2 \end{bmatrix}$$

On premultiplying the eqn (B) by Z^{-1} we get,

$$\begin{bmatrix} I_{a_0} \\ I_{a_1} \\ I_{a_2} \end{bmatrix} = \begin{bmatrix} 1/Z_0 & 0 & 0 \\ 0 & 1/Z_1 & 0 \\ 0 & 0 & 1/Z_2 \end{bmatrix} \begin{bmatrix} 0 \\ E_a \\ 0 \end{bmatrix} - \begin{bmatrix} 1/Z_0 & 0 & 0 \\ 0 & 1/Z_1 & 0 \\ 0 & 0 & 1/Z_2 \end{bmatrix} \begin{bmatrix} E_a - I_{a_1} Z_1 \\ E_a - I_{a_1} Z_1 \\ E_a - I_{a_1} Z_1 \end{bmatrix}$$

$$\begin{bmatrix} I_{a_0} \\ I_{a_1} \\ I_{a_2} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{E_a}{Z_1} \\ 0 \end{bmatrix} - \begin{bmatrix} \frac{E_a - I_{a_1} Z_1}{Z_0} \\ \frac{E_a - I_{a_1} Z_1}{Z_1} \\ \frac{E_a - I_{a_1} Z_1}{Z_2} \end{bmatrix}$$

On multiplying,

$$I_{a_0} = -\left(\frac{E_a - I_{a_1} Z_1}{Z_0}\right) = -\frac{E_a}{Z_0} + \frac{I_{a_1} Z_1}{Z_0} \longrightarrow \mathrm{I}$$

$$I_{a_1} = \frac{E_a}{Z_1} - \left(\frac{E_a - I_{a_1} Z_1}{Z_1}\right) = \frac{I_{a_1} Z_1}{Z_1} = I_{a_1} \longrightarrow \mathrm{II}$$

$$I_{a_2} = -\left(\frac{E_a - I_{a_1} Z_1}{Z_2}\right) = -\frac{E_a}{Z_2} + \frac{I_{a_1} Z_1}{Z_2} \longrightarrow \mathrm{III}$$

Here $I_a = 0$, On expressing I_a by its symmetrical components we get,

$$I_a = I_{a0} + I_{a1} + I_{a2} = 0 \longrightarrow \text{IV}$$

On sub for I_{a0}, I_{a1} & I_{a2} from eqns I, II, III in IV

$$-\frac{E_a}{Z_0} + \frac{I_{a1}Z_1}{Z_0} + I_{a1} - \frac{E_a}{Z_2} + \frac{I_{a1}Z_1}{Z_2} = 0$$

$$I_{a1}\left(\frac{Z_1}{Z_0} + 1 + \frac{Z_1}{Z_2}\right) - E_a\left(\frac{1}{Z_0} + \frac{1}{Z_2}\right)$$

$$I_{a1}\left(1 + Z_1\left(\frac{1}{Z_0} + \frac{1}{Z_2}\right)\right) = E_a\left(\frac{1}{Z_0} + \frac{1}{Z_2}\right)$$

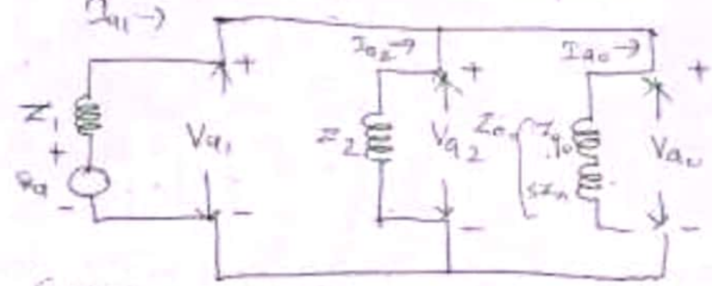
$$I_{a1}\left(1 + Z_1\left(\frac{Z_0+Z_2}{Z_0Z_2}\right)\right) = E_a\left(\frac{Z_0+Z_2}{Z_0Z_2}\right) \longrightarrow \text{V}$$

On multiplying the eqn V throughout by $\frac{Z_0Z_2}{Z_0+Z_2}$ we get

$$I_{a1}\left(\frac{Z_0Z_2}{Z_0+Z_2} + Z_1\right) = E_a$$

$$\therefore I_{a1} = \frac{E_a}{Z_1 + \frac{Z_0Z_2}{Z_0+Z_2}} \longrightarrow \text{VI}$$

Using this eqn, the equivalent ckt of (G) under double line to ground fault can be drawn.



Connection of the seq n/ws of an unloaded (G) for a double line to ground fault on phase b & c

Under this fault condition, the seq n/ws should be connected in ||el since the +ve, -ve & zero seq voltages are equal during this fault.

In the absence of a ground connection at the (G) no ct can flow into the gnd at the fault. In this case Z_0 would be infinite and I_{a0} would be zero & so the fault will be similar to line to line fault.

Here the fault ct, $I_f = I_b + I_c$

To determine the fault ct, first the sym components I_{a1}, I_{a0} & I_{a2} are calculated using eqns

V, I & III. These eqns are presented here for convenience.

$$I_{a1} = \frac{E_a}{Z_1 + \frac{Z_0 Z_2}{Z_0 + Z_2}} \; ; \; I_{a0} = \frac{-E_a}{Z_0} + \frac{I_{a1} Z_1}{Z_0}$$

$$I_{a2} = -\frac{E_a}{Z_2} + \frac{I_{a1} Z_1}{Z_2}$$

Alternatively I_{a1} can be calculated using V, then I_{a2} & I_{a0} can be calculated using ct division rules.

$$I_{a2} = -I_{a1} \times \frac{Z_0}{Z_0 + Z_2} \quad \&$$

$$I_{a0} = -(I_{a1} - I_{a2})$$

The unbalanced cts I_a, I_b & I_c are related to the sym. components of cts by the following eqn.

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

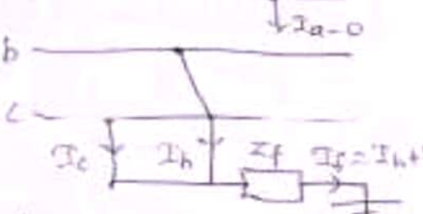
From this eqn we get

$$I_a = I_{a0} + I_{a1} + I_{a2}$$
$$I_b = I_{a0} + a^2 I_{a1} + a I_{a2}$$
$$I_c = I_{a0} + a I_{a1} + a^2 I_{a2}$$

$\therefore$ fault ct, $I_f = I_b + I_c$

$$= I_{a0} + a^2 I_{a1} + a I_{a2} + I_{a0} + a I_{a1} + a^2 I_{a2}$$
$$= 2 I_{a0} + (a + a^2) I_{a1} + (a + a^2) I_{a2}$$
$$I_f = 2 I_{a0} + (a + a^2)(I_{a1} + I_{a2})$$

Fault thro an impedance :-



Connection dia of stubs for a double line to grd fault thro the an impedance

A double line to ground fault at point F in a power sm, the a fault imp Z_f can be represented by connecting three stubs.

The ct & voltage conditions at the fault are

$$I_a = 0 \rightarrow 1$$

$$V_b = V_c = Z_f (I_b + I_c) \rightarrow 2$$

The line cts are gn by

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \rightarrow 3$$

From row 1 of eqn we get,

$$I_a = I_{a0} + I_{a1} + I_{a2}$$

From eqn 1 we get, $I_a = 0$

$$\therefore I_{a0} + I_{a1} + I_{a2} = 0$$

$$I_{a0} = -(I_{a1} + I_{a2}) \longrightarrow 4$$

From row 2, 3

$$I_b = I_{a0} + a^2 I_{a1} + a\, I_{a2}$$

$$I_c = I_{a0} + a\, I_{a1} + a^2 I_{a2}$$

$$I_b + I_c = 2I_{a0} + (a^2+a)\, I_{a1} + (a^2+a)\, I_{a2} \rightarrow 5$$

We know that,

$$1 + a + a^2 = 0 \qquad \therefore a + a^2 = -1 \rightarrow 6$$

From eqns 5 & 6

$$I_b + I_c = 2I_{a0} - I_{a1} - I_{a2}$$

$$= 2I_{a0} - (I_{a1} + I_{a2}) \longrightarrow 7$$

From eqn 4

$$I_b + I_c = 2I_{a0} - (-I_{a0}) = 3I_{a0} \rightarrow 8$$

The sym. components of voltages after substituting $V_c = V_b$ are gn by

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

from row 1,

$$V_{a0} = \frac{1}{3}(V_a + V_b + V_c) = \frac{1}{3}(V_a + 2V_b) \rightarrow (9)$$

From row 2 & 3

$$V_{a1} = V_{a2} = \frac{1}{3}(V_a + aV_b + a^2 V_b)$$

$$= \frac{1}{3}(V_a + (a+a^2)V_b)$$

$$= \frac{1}{3}(V_a - V_b) \rightarrow (10) \qquad (\because a + a^2 = -1)$$

From eqn 9 & 10 we can write,

$$V_{a0} - V_{a1} = \frac{1}{3}(V_a + 2V_b) - \frac{1}{3}(V_a - V_b)$$

$$= \frac{1}{3}(\cancel{V_a} + 2V_b - \cancel{V_a} + V_b) = V_b \longrightarrow (11)$$

On substituting for V_b from eqn 2 in 11

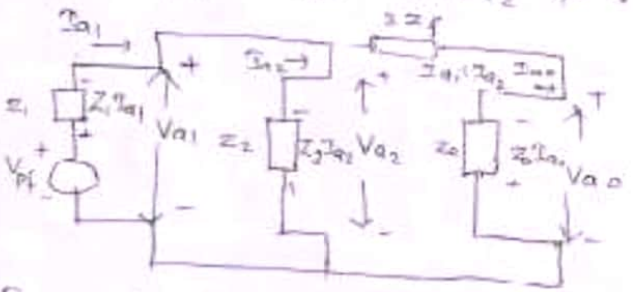
$$V_{a0} - V_{a1} = Z_f (I_b + I_c)$$

On substituting $(I_b + I_c)$ from eqn 8

$$V_{a0} - V_{a1} = Z_f \, 3 I_{a0}$$

$$\therefore V_{a0} = V_{a1} + 3Z_f I_{a0} \rightarrow 12$$

Also $V_{a0} = V_{a2} + 3 Z_f I_{a0} \rightarrow 13$



From fig $I_{a1} = \dfrac{V_{pf}}{Z_1 + \dfrac{Z_2(Z_0 + 3Z_f)}{Z_2 + Z_0 + 3Z_f}} \rightarrow 14$

$$V_{a1} = V_{pf} - Z_1 I_{a1} \rightarrow 15$$

$V_{a2} = V_{a1}$; $I_{a2} = -\dfrac{V_{a2}}{Z_2}$; $I_{a0} = -(I_{a1} + I_{a2}) \rightarrow 16$

From fig (1) & eqn 8

Fault ct, $I_f = I_b + I_c = 3 I_{a0}$.